Quantitative Entropy Study of Language Complexity

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We study the entropy of Chinese and English texts, based on characters in case of Chinese texts and based on words for both languages. Significant differences are found between the languages and between different personal styles of debating partners. The entropy analysis points in the direction of lower entropy, that is of higher complexity. Such a text analysis would be applied for individuals of different styles, a single individual at different age, as well as different groups of the population.

I. INTRODUCTION

Sustainable development was first addressed by Erwin Schrödinger [1] based on entropy, where development was characterized by increasing "orderliness" (nowadays complexity). He pointed out that the development of highly complex forms of matter (or life) should be built on less complex forms. This means decreasing entropy, while increasing the entropy of matter should be avoided if we want to maintain sustainable development.

Recently the build up of complexity on the example of 1 kg matter in different forms was studied quantitatively, starting from the simplest example of ideal gases, and then continue with more complex chemical, biological, and living structures [2]. The complexity of these systems was assessed quantitatively, based on their entropy. We use the method introduced in Ref. [2], which attributed the same entropy to known physical systems and to complex organic molecules up to most complex Human Genome DNA.

Schrödinger [1] has also discussed and concluded that the emergence of life does not require new fundamental laws of physics, which allow for non-increasing entropy. Actually, as the Earth is an open system [3], with a boundary condition strongly decreasing entropy, this boundary condition enforces development towards decreasing entropy, i.e. increasing complexity.

The Human brain has a vastly superior possibility of complexity, than biological molecules, and it carries abstract information, as well as many vegetative and reflex functions. The direct calculation of the complexity of the coding in the Human neural network is beyond our present knowledge, but we can make studies of the stored, consciously reachable, information and its complexity.

The conscious thinking can be indirectly studied via the analysis of Human languages. We can think about one subject at a time, just like we can speak about one subject at a time.

II. LANGUAGE COMPLEXITY

As discussed in Ref. [2], to analyze the system from entropy or complexity point of view we have to consider two basic aspects: (i) the quantum of information or of the substance we analyse and (ii) the possibility of all configurations in a set of degrees of freedom, as well as the realized, realizable or existing configurations from the set of all possible configurations.

Regarding the first point (i) in physics we quantized the phase space (the six dimensional position and momentum space) and have introduced the volume of the phase space element based on the quantum mechanical uncertainty relation.

In case of a language the basic element could be the word. This can also be the basic element of the conscious thinking. At this time we do not have sufficient information on how a "word" is represented in a neural network, how many neurons and synapses are involved, and what is the weight of the corresponding material. Hopefully in the future we can acquire the knowledge to reply to these questions. This situation is similar to the early development of statistical physics, when kinetic theory and thermodynamics were already known, with entropy and the second law of thermodynamics. At this time it was already realized that the phase space should be quantized, but before the quantum mechanics one did not know what should be this phase space volume. This did lead to a state where entropy was only defined up to a constant, which could be chosen free. Still similar systems could be analysed quantitatively, and compared to each other.

When we chose the word as the quantum of a language we are in a similar situation as the early thermodynamics. A constant is *remaining to be determined*, to compare the entropy of the language to that of the ideal gases or the Human DNA sequence.

The second condition (ii) is not very problematic in case of a language, the given amount of words can be determined by analysis of texts. Then the number of all possible configurations can be calculated for any given length sentence. For long sentences this number of configurations can become astronomically high, but one can analyse the distribution of the lengths of sentences as well as the maximal length. This will make the number of possible configurations finite. Subsequently one can analyse existing texts and can evaluate the number of existing configurations. This last step, can be done for a single person's language (who wrote extensively, so that we can analyse his or her language). It can be done for writings in a region where the language is used, or for all users of the language.[1]

III. ANALYSIS OF CHINESE LANGUAGE WITH CHARACTERS

As a first example we use the analysis of the Chinese language to have an order of magnitude estimate of the quantitative complexity or entropy of Human thinking via the language.

The Chinese language uses characters. On average a person uses about 3000 characters in communication. The characters may form words of one, two or more characters. These afterwards, form sentences, which are separated by periods (and exclamation or question marks) in writing.

Texts of about 26000-80000 Chinese characters were analysed in four samples, Sample *I* to *IV* [4–7]. We evaluated how many different Chinese characters were contained in a given sample, N_c. Then in the first evaluation, we checked how many one character sentences, N_1^c, two character sentences, N_2^c, three character sentences, N_3^c, and so on, up to 35 character sentences were in the samples. See table I.

Sample	N_s	N_c	N_1^c	N_2^c	N_3^c	N_4^c	N_5^c	N_6^c	N_7^c	N_8^c	N_9^c
I	79959	2553	163	375	248	225	209	193	168	195	149
II	79470	2137	69	130	100	126	123	181	170	156	169
III	26671	2096	4	4	5	6	24	20	32	27	30
IV	29083	1916	1	4	5	20	19	18	29	38	48

TABLE I. Number of all Chinese characters, N_s, and of different Chinese characters, N_c, in the Sample texts, *I* to *IV* are shown. Then the sentences (separated by periods) are analysed: the one character sentences, two character sentences, and so on. The number of different k-character sentences, N_k^c were counted in the sample texts. The longest sentences were between 162, 119, 145, and 129 characters for the four samples respectively.

Let us now consider the two character sentences. This can be formed by choosing one character of the N_c for the first position, and another from the N_c for the second position. The two characters may be identical and the sequence of the characters is meaningful. Consequently the maximum number of possible *two character sentences* is N_c^2, and the probability of one configuration is $p_i = 1/N_c^2$. Thus the maximum entropy of all possible two character sentences is

$$H(X_2^{max}) = -\sum_{all} p_i \ln p_i = -N_c^2 \frac{1}{N_c^2} \ln \frac{1}{N_c^2}$$
$$= \ln N_c^2 = 15.690, 15.334, 15.296, 11.116 \ , \tag{1}$$

for Samples *I*, *II*, *III*, *IV*, respectively.

In real physical or biological situations not all (hypothetical) configurations are realized. The number of observed or Realized (R) different *two character sentences* for Sample *I* is only $N_2^c = 375$. Consequently the corresponding specific configuration entropy is

$$H(X_2^R) = -\sum_{i=1}^{N_2^c} p_i \ln p_i = -N_2^c \frac{1}{N_c^2} \ln \frac{1}{N_c^2}$$
$$= 2N_2^c \ln(N_c)/N_c^2 = 9.027 \cdot 10^{-4}. \tag{2}$$

This entropy is proportional with the number of two-character sentences, N_2^c. At the same time N_2^c is also proportinal with the size of the Sample text.

We can do the same analysis in this Sample *I* text, for sentences of one, three, four, etc., ..., 162 Chinese character sentences. These are unrelated configurations and as the entropy is additive the specific entropy of *all sentences* of Sample text *I*, based on Chinese characters is

$$\begin{aligned}\sigma_c &= H(X_1^R) + H(X_2^R) + H(X_3^R) + H(X_4^R) + ... \\ &= 5.009 \cdot 10^{-1} + 9.027 \cdot 10^{-4} + 3.508 \cdot 10^{-7} + ... \\ &= 5.018 \cdot 10^{-1}.\end{aligned} \tag{3}$$

One can see that the few (one, two, three) character sentences provide the largest contribution to the entropy, and the longer ones have minor contribution. The higher level of complexity is achieved by minimizing the use of one or two character sentences. The very long sentences have very large number of hypothetical possibilities, while occur very seldom in the text. The contribution of 10 character sentences to the entropy is $\sigma_{10c} < 10^{-30}$, and the longer ones are even smaller. One could take into account the relative frequencies of the different length sentences, but the relative frequencies of long sentences in the sample texts is also rapidly decreasing. Therefore their entropy contribution is utterly negligible. This also indicates the hint that the length of the Sample text is not very important beyond some number as it leads to relatively small change in the results.

The entropy of a Sample text is proportional to the length of the text. In order to compare texts of different lengths we can introduce a specific entropy for 10000

[1] In some languages the computational analysis of texts may be problematic, e.g., in Hungarian the form of a word in a given text is changing to the extent that it is not possible to find the root of a word in a dictionary. The good knowledge of the language and grammar would be necessary to do this analysis, what computational analysis programs cannot do at this time.

Sample	N_s	σ_c	$\sigma_{c/10k}$
I	79959	$5.018 \cdot 10^{-1}$	$6.276 \cdot 10^{-2}$
II	79470	$2.480 \cdot 10^{-1}$	$3.121 \cdot 10^{-2}$
III	26671	$1.461 \cdot 10^{-2}$	$5.477 \cdot 10^{-3}$
IV	29083	$3.961 \cdot 10^{-3}$	$1.362 \cdot 10^{-3}$

TABLE II. Specific entropy of the Sample texts based on Chinese characters, where N_s is the number of characters in the Sample text, σ_c is the entropy of the text, and $\sigma_{c/10k}$ is the entropy of the text normalized to 10000 character length.

characters (or words), so for Sample I:

$$\sigma_{c/10k} \equiv 10000 \cdot \sigma_c / N_s == 6.276 \cdot 10^{-2} \ . \qquad (4)$$

Samples $I - IV$ have a different texts with different parameters. The entropy analysis can be preformed the same way as for Sample I, resulting:

$$\sigma_{c/10k} = (6.276,\ 3.121,\ 0.5477,\ 0.1362) \cdot 10^{-2} \ , \qquad (5)$$

for Sample texts $I - IV$ respectively. The shorter text samples have a tendency to give smaller length normalized specific entropy. The results are summarized in Table II.

IV. ANALYSIS OF CHINESE LANGUAGE WITH WORDS

In the Chinese language, although single characters may correspond to a word, certain two or three character combinations are unique and can be considered as words. So in this sense words can be considered as the basic parts of a sentence instead of Chinese characters. See table III.

Sample	N_s	N_w	N^w_1	N^w_2	N^w_3	N^w_4	N^w_5	N^w_6	N^w_7	N^w_8	N^w_9
I	49835	10122	558	304	348	279	282	283	249	257	241
II	47911	8169	208	174	219	268	260	264	273	262	261
III	16780	5086	5	13	22	38	46	41	53	55	54
IV	18501	4775	4	13	19	55	50	47	68	65	49

TABLE III. Number of all Chinese words, N_s, and the different Chinese words, N_w, in the Sample texts, $I - IV$ are shown. Then the sentences (separated by periods) are analysed: the one word sentences, two word sentences, and so on. The number of different k-word sentences, N^w_k were counted in the sample texts.

In Chinese writing the words are not separated by spaces, but commas, quotation marks and other punctuation may separate words. We employ the package "jiebaR" with *R language* to distinguish the words.

We can calculate the maximum specific entropy for all hypothetical k-word combinations, using the number of different Chinese words in the sample text. For example for *two word sentences*

$$H(X_2^{max}) = -\sum_{all} p_i \ln p_i = -N_w^2 \frac{1}{N_w^2} \ln \frac{1}{N_w^2}$$
$$= \ln N_w^2 = 18.444, 18.016, 17.068, 14.192 \ , \qquad (6)$$

for Samples I, II, III, IV, respectively. These are smaller than the max entropies for Chinese characters as the observed number of words is smaller than the number of characters.

The number of observed *two word sentences* in the Sample texts is of course much smaller, than the hypothetical maximum, thus the specific entropy for two word sentences is also smaller:

$$H(X_2^R) = (508.4,\ 229.4,\ 8.390,\ 7.096) \cdot 10^{-3}. \qquad (7)$$

for Samples I, II, III, IV, respectively.

Then we add up the entropy contribution of *all observed sentences* of all lengths in the Sample texts. This provides the total specific entropy

$$\sigma_w = (508.5,\ 229.4,\ 8.399,\ 7.106)\ \cdot 10^{-3}, \qquad (8)$$

for Sample texts $I - IV$ respectively. We summarize these data in Table IV.

Sample	N_s	σ_w	$\sigma_{w/10k}$
I	49835	$5.085 \cdot 10^{-1}$	$1.020 \cdot 10^{-1}$
II	47911	$2.294 \cdot 10^{-1}$	$4.788 \cdot 10^{-2}$
III	16780	$8.399 \cdot 10^{-3}$	$5.005 \cdot 10^{-3}$
IV	18501	$7.106 \cdot 10^{-3}$	$3.841 \cdot 10^{-3}$

TABLE IV. Specific entropy of the Sample texts based on Chinese words, where N_s is the number of words in the Sample text, σ_w is the entropy of the text, and $\sigma_{w/10k}$ is the entropy of the text normalized to 10000 word length.

The Entropies obtained from the analysis of the words is similar to those that were based on characters. The difference between the entropy and the length normalized entropy based on characters and words is smaller in the case of words.

V. ANALYSIS OF ENGLISH TEXTS

The analysed English text samples [8–11], contained 102613, 93668, 6480, and 8992 words. The 3rd and 4th texts are from the first presidential candidacy debate of Hillary Clinton and Donald Trump. See table V.

Noticeable that while most Chinese sentences have 10 words or less the in the analysed English text most sentences have about 20 words! This has an interesting effect on the complexity or entropy analysis of the text.

Sample	N_s	N_w	N_1^w	N_2^w	N_3^w	N_4^w	N_5^w	N_6^w	N_7^w	N_8^w
I	102613	7966	0	6	33	68	155	120	189	272
II	93668	5745	44	10	12	11	7	8	28	35
III	6480	1309	2	6	7	19	29	28	22	28
IV	8992	1225	18	12	27	42	74	76	66	64

TABLE V. Number of all English words, N_s, and of different English words, N_w, in the Sample texts. Then the sentences (separated by periods) are analysed: the two word sentences, three word sentences and so on. The number of different k-word sentences, N_k^w were counted in the sample texts. While Samples *I* and *II* are extended written texts, *III* and *IV* are debates of Hillary Clinton and Donald Trump, respectively. The debate texts are shorter and thus also their vocabulary is more constrained.

We can calculate the maximum specific entropy for all hypothetical k-word combinations. For example for *two word sentences*

$$H(X_2^{max}) = 17.876,\ 17.312,\ 14.354,\ 14.222, \quad (9)$$

for the three English Sample texts. This is larger than the max entropies for Chinese word texts, due to the larger number of words in the text. See Figure 1.

The number of observed two word sentences in the text is of course much smaller, than the hypothetical maximum, thus the specific entropy for realized *two word sentences* is also smaller

$$H(X_2^R) = 8.494{\cdot}10^{-7},\ 5.245{\cdot}10^{-6},\ 5.026{\cdot}10^{-5},\ 1.137{\cdot}10^{-4}, \quad (10)$$

for the English text Sample.

Then we add up the entropy contribution of all observed sentences of all lengths. This provides the total specific entropy for *all sentences* for the four English Sample texts. See Table VI.

Sample	N_s	σ_w	$\sigma_{w/10k}$
I	102613	$8.499 \cdot 10^{-7}$	$8.283 \cdot 10^{-8}$
II	93668	$6.630 \cdot 10^{-2}$	$7.078 \cdot 10^{-3}$
III	6480	$1.102 \cdot 10^{-2}$	$1.700 \cdot 10^{-2}$
IV	8992	$1.046 \cdot 10^{-1}$	$1.163 \cdot 10^{-1}$

TABLE VI. Specific entropy of the Sample texts based on English words, where N_s is the number of words in the Sample text, σ_w is the entropy of the text, and $\sigma_{w/10k}$ is the entropy of the text normalized to 10000 word length.

For the 1st Sample text, this is the same as that of the shortest 2-word sentences in the text, because the next longer 3-word sentences have an entropy value that is 4 orders of magnitude smaller. Due to the lack of single word sentences, and the small number of two word sentences the entropy of the English text is much smaller than that of the Chinese texts. In the much shorter debate texts of Clinton and Trump the number of very short sentences dominates. Trump has a much larger number of short sentences and this increases the total entropy of his text, contrary to the fact that the number of the words in his text is significantly larger. See Figure 2.

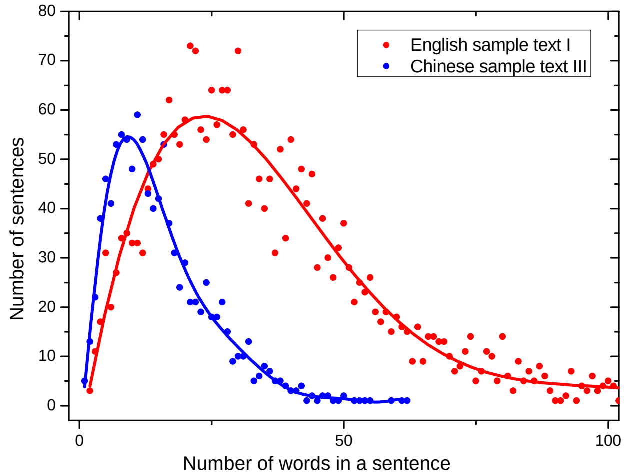

FIG. 1. (Color online) The distribution of the sentences according to their length. The length is measured by the number of words in a sentence, while the number of sentences of a given length in a Sample text is shown. The red dots correspond to the English Sample text I, peaking at ~ 26 words, while the blue dots to the Chinese Sample text III, peaking at ~ 8 words. The lines are to guide the eye.

VI. DISCUSSION

The analysis of the complexity or entropy of languages can of course be used for comparing different languages, or different texts, or authors to each other. There is a vast amount of literature analysing languages with many different methods. Here we have chosen a relatively simple, and transparent method.

But the entropy value as a general feature of material can actually lead to conclusion regarding the entropy of the physical and biological structure of the brain, and the information content in abstract sense. The language can be representative of the conscious operation of the brain. The physical and biological complexity has to be much larger, as the brain is responsible for the vegetative operation of the nervous system as well as the dynamical changes of the operation and the human activity. The language itself is just a static set of information, but it has to be learned, so it is a structure, which is the product of training or learning. The language itself can characterize the development, see e.g. [12].

The language can also be attributed to a given amount

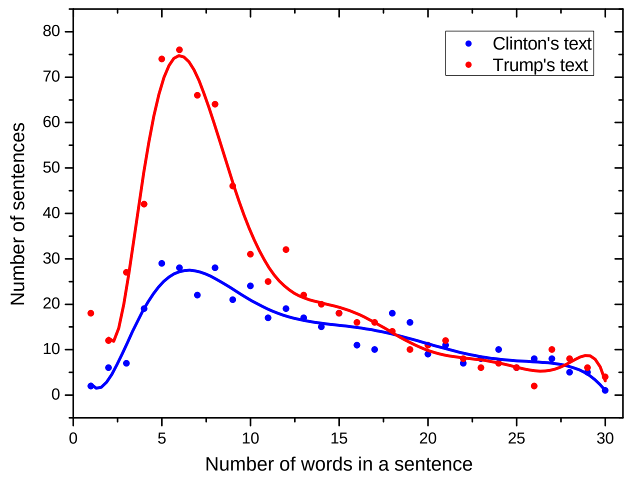

FIG. 2. (Color online) The distribution of the sentences according to their length for the first presidential debate between Hillary Clinton and Donald Trump. The red dots correspond to Trump's text, while the blue dots to Clinton's. Trump's text is dominated by short sentences peaking with 76 sentences of 6 word length. The lines are to guide the eye.

of material. It is a given part of the brain, even if we cannot identify it. Plausibly the same part of the brain carries other static information as well as dynamical information also. This way in addition to the specific entropy of the language, σ_c or σ_w we can also estimate the physical entropy S_{1kg} or at least a lower limit of it.

In case of usual (Shannon) entropy estimates the normalization is not the same as the physical one, but it is perfectly sufficient for comparative studies of these type of structural entropies.

In this analysis the role of physical phase-space or configuration space is taken over by the "word-space" or "Chinese character-space". These spaces could in principle be extended to infinity, but in fact taking all words of a language in a historical period the word-space of a language is finite. This is also necessary as the language is a means of communication. Thus, we cannot add up the word-space of all languages.

VII. CONCLUSIONS

We have demonstrated quantitatively the increasing complexity of materials, and used the entropy for unit amount of material to be able to get a measure. This idea stems from Ervin Schroedinger, but our knowledge today makes it possible to extend the level of quantitative discussion to complex live materials.

We may continue these studies to higher levels of material structures, like living species, artificial constructions, symbiotic coexistence of different species, or grouping of the same species. up to even structures in Human society.

The main achievement of the earlier work [2] was to show how the entropy in the physical phase space and the entropy of structural degrees of freedom(Shannon entropy) can be discussed on the same platform. For further developments it is important to point out two fundamental aspects of the entropy concept: (i) the *quantization* of the space of a given degree of freedom, and (ii) the *selection of the realized, realizable or beneficial configurations* from all the possible ones.

In the present work we introduced a quantization as the number of words or Chinese characters. At this moment of time we do not know how to relate this quantization to the basic physical quantum of the occupation of an elementary phase-space element. Thus, the relative normalization of the quantitative complexity or entropy of the language is still missing. We would need a much more detailed knowledge about the representation of language in the neural network of the human brain.

The other aspect of the entropy calculation is actually solved in case of the human language or languages, as the realized configurations can be relatively easily determined by the analyses of the texts.

The Sample text examples presented are all static point. As we see on the example of the nervous system the dynamical change of the entropy of the system is also important. The text analysis could trace down the change of the complexity of the texts of an individual, which could be a measure of the period of increasing complexity at early years compared to decreasing complexity and increasing entropy later. Such analysis could be performed on the novels of authors, who were active for many years. The dynamics and direction of these changes is also essential as shown in Ref. [13].

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